of heat-transfer agent; F, area of cross section; V, volume; α, heat-removal coefficient; q, heat-flux denxity. Indices: h, heater; c, condensation; T, transport; w, wick; eff, effective; t, theory, ex, experimental; wall, wall.

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HEAT TRANSFER IN A TOROIDAL VESSEL PARTLY FILLED WITH LIQUID

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UDC 536.242

Measurements are reported on heat transfer for a toroidal vessel partly filled with liquid oxygen, nitrogen, or hexane for a constant heat-flux density at the boundary.

Long-term storage of cryogenic liquids has made it necessary to study processes due to external heat leaks that occur in vessels filled with such liquids. The final result from such studies should be recommendations on calculating the temperature distribution and pressure increase in such a vessel.

Here we report studies on the heat transfer in a toroidal vessel filled with liquid oxygen, nitrogen, or hexane for a constant heat-flux density at the casing.

The measurements were made on two toroidal vessels in which the small and large radii of the toroids were, respectively, $r_1 = 0.072$ m, $r_2 = 0.125$ m and $r_1 = 0.096$ m, $r_2 = 0.175$ m; the major process parameters were varied over the following ranges: heat-flux density $q = 15-300$ W/m^2; reduced initial temperature $T_0/T_c = 0.46-0.64$; and liquid filling factor $m = V_l/V = 0.17-0.90$. The working technique has previously been described [1].

The process occurs in the following sequence [2, 3]. When the heat flux begins to be supplied to the wall–liquid boundary, we obtain a boundary layer within which the liquid moves along the heated wall upward to the phase interface. The rising currents produce eddies near the surface of the liquid, which involve downward liquid flow. The flow is closed within the body of the liquid. The time τ_* needed to produce the first closed cycle has been interpreted [3] as the time needed to complete stage I (the internal transitional state). We now consider the details of the temperature distribution for each of the successive states. Figure 1 shows the temperature difference between the free surface and the lower region of uniform temperature as a function of time, $\vartheta_s - \vartheta_0 = f(\tau)$; the initial section is close to linear, while the latter section follows the law $\vartheta_s - \vartheta_0 \sim \tau^{0.5}$, which is ultimately followed by a horizontal part. Figure 1a shows that the rate of development of the temperature differential decreases as time passes. A comparison has been made [4] of the time for transition from the linear part $\vartheta_s - \vartheta_0 = f(\tau)$ to the $\vartheta_s - \vartheta_0 \sim \tau^{0.5}$ part with the time τ_* [3]. The relationship between these times indicates that the instant of transition from the linear part of $\vartheta_s - \vartheta_0 = f(\tau)$ to $\vartheta_s - \vartheta_0 \sim \tau^{0.5}$ represents the boundary between modes I and II, namely, τ_*. The instant τ_{**} of transition from $\vartheta_s - \vartheta_0 \sim \tau^{0.5}$ to $\vartheta_s - \vartheta_0 = \text{const}$ can be considered as the boundary between states II and III, which corresponds to the onset of a quasistationary state.

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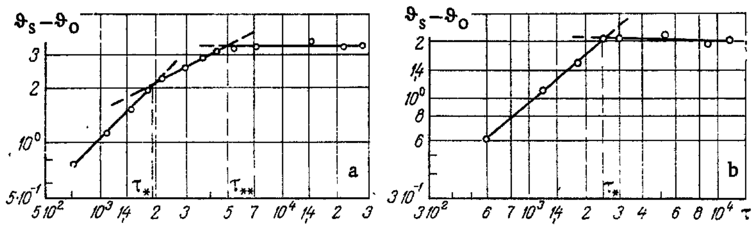

Fig. 1. Maximal vertical temperature difference in the liquid as a function of time at various stages: a) $r_1 = 0.072$ m, $m = 0.87$, $q = 28.3$ W/m^2, nitrogen $T_0 = 77.2$ °K; b) $r_1 = 0.072$ m, $m = 0.32$, $q = 28.3$ W/m^2, nitrogen $T_0 = 77.2$°K; τ, sec; $(\vartheta_s-\vartheta_0)$, deg.

If only a small fraction of the volume is filled ($m \leq 0.5$), there is no part where $\vartheta_s-\vartheta_0 \sim \tau^{0.5}$ applies (Fig. 1b).

We consider the thermodynamic relations that reflect the state of the system as a whole [5] in order to define a system of similarity criteria that describe the process; we combine these equations with the equations of motion and energy. In the internal transitional state, the temperature distribution in the liquid is close to isothermal, and the maximum deviation from the saturation temperature near the bottom does not exceed 4%. From the thermodynamic viewpoint, the process is of isochoric type occurring under conditions close to equilibrium. In that case, the relevant equation is

$$\delta Q = dU = MC_x dT_s, \tag{1}$$

where $C_x = C' + x(C'' - C')$ is the heat capacity of the system containing wet vapor and M is the equivalent mass of the system.

We rewrite (1) in the following form [6]:

$$qS = MC_x \frac{dT_s}{d\tau} = V\rho_e (C' + x(C'' - C')) \frac{dT_s}{d\tau} =$$

$$= V\rho_e \left[C_p + \frac{(1-m)\rho''}{\rho_e} \left(\frac{\partial L}{\partial T_s} - \frac{L}{T_s} \right) \right] \frac{dT_s}{d\tau}. \tag{2}$$

In (2), L, C_x, and ρ_e refer to the initial temperature in the system T_0, the last of these being the equivalent density $\rho_e = m\rho' + (1-m)\rho''$, while the specific heat of the liquid on the saturation line is taken as approximately equal to the specific heat at constant pressure, $C' \cong C_p$ [6]. Reduction of (2) to dimensionless form gives us a series of dimensionless relations:

$$\mathrm{Ho}_* = \frac{q\tau_* S}{VC_p m\rho' T_0}, \quad \frac{x(C'' - C_p)}{C_p}, \quad \bar{\tau} = \frac{\tau}{\tau_*}, \quad \bar{\vartheta} = \frac{\vartheta}{T_0}. \tag{3}$$

We take T_0 as the scale for the temperature since it reflects the initial state of the system, while the scale for the time is τ_*, namely, the time needed to complete stage I. If we neglect the mass of the vapor in ρ_e and put $\rho_e \cong m\rho'$, we can convert (3) to

$$\mathrm{Ho}_* = \frac{q\tau_*}{lmC_p\rho' T_0}, \quad \frac{(1-m)\rho''}{m\rho' C_p} \left(\frac{\partial L}{\partial T_s} - \frac{L}{T_s} \right), \quad \bar{\tau}, \ \bar{\vartheta}. \tag{3a}$$

It follows from (3) or (3a) that the wall temperature as a function of time in stage I can be put as

$$\bar{\vartheta} = f \left[\frac{\tau}{\tau_*}, \ \mathrm{Ho}_*, \ \frac{x(C'' - C_p)}{C_p} \right]. \tag{4}$$

The relative time for production of the circulation $\mathrm{Fo}_* = a\tau_*/l^2$ can be expressed as a function of the generalized variables obtained by reducing the transport equations to dimensionless form in conjunction with the uniqueness conditions. The following general relationship for Fo_* was derived by processing the experimental data:

$$\mathrm{Fo}_* = f \left[\mathrm{Ra}^*, \ \mathrm{Pr}, \ m, \ \frac{\rho_e}{\rho_c} \right], \tag{5}$$

where ρ_e/ρ_c is a parametric criterion reflecting the thermodynamic state of the system.

It follows from (4) and (5) that the following similarity numbers are the most important in describing the temperature distribution:

$$\mathrm{Ra}^* = \frac{ql^4 g\beta}{\nu a\lambda},\quad \mathrm{Pr} = \frac{\nu}{a},\quad \frac{x(C''-C_p)}{C_p},\quad m,\quad \frac{\rho_e}{\rho_c},$$

$$\mathrm{Ho}_* = \frac{q\tau_*}{mlC_p\rho' T_0},\quad \vartheta = \frac{\vartheta}{T_0},\quad \bar{\tau} = \frac{\tau}{\tau_*},\quad K_t = \frac{m\,\mathrm{Ho}_*}{\mathrm{Fo}_*} = \frac{ql}{\lambda T_0}, \tag{6}$$

where $K_t = ql/\lambda T_0$ is a parametric criterion representing the ratio between the temperature scales for the heat transfer (ql/λ) and the thermodynamic temperature T_0. Then K_t allows us to transform (6) to

$$\mathrm{Ra}^* = \frac{ql^4 g\beta}{\nu a\lambda},\quad \mathrm{Pr} = \frac{\nu}{a},\quad \frac{x(C''-C_p)}{C_p},\quad m,\quad \frac{\rho_e}{\rho_c}, \tag{6a}$$

$$\mathrm{Fo}_* = \frac{a\tau_*}{l^2},\quad \bar{\vartheta} = \frac{\vartheta\lambda}{ql},\quad \bar{\tau} = \mathrm{Fo} = \frac{a\tau}{l^2},\quad K_t = \frac{ql}{\lambda T_0}.$$

The system (6) or (6a) for the initial stage was then used to process the data for the later stages; one is justified in such processing on account of the universal relationship

$$\frac{\vartheta}{\vartheta_*} = f\left(\frac{\tau}{\tau_*},\ \frac{x(C''-C_p)}{C_p}\right), \tag{7}$$

where ϑ_* is the temperature of the liquid at time τ_*. Figure 2 shows an example of this relationship for the surface temperature of the liquid. The results in Fig. 2 relate to the range $\mathrm{Fo} = 10^{-3}$–10^{-1}, which covers all three time intervals. We see from (7) that the temperature at any time can be represented as a function ϑ_*, τ_*, and $x(C''-C_p)/C_p$. However, ϑ_* and τ_* relate to the initial stage and are generalized functions of the similarity numbers of (6) in relative form, and one therefore supposes that the temperature in the subsequent stages can be described by means of the generalized variables for the initial stage. All parameters of the process, including the properties of the medium, are referred to the initial instant $\tau = 0$ in the similarity numbers, since the maximum change in temperature from the initial value does not exceed 10% at that stage.

The measurements were processed to give generalized relationships that allow one to calculate the time course of the surface temperature and the limiting times corresponding to the various stages. The kinks in $\vartheta_s - \vartheta_0 = f(\tau)$, as in Fig. 1, were used to define the boundaries.

The following expression applies for the boundary between states I and II:

$$\mathrm{Fo}_* = K\,\mathrm{Ra}^{*n_1}\,\mathrm{Pr}^{n_2}(1-m)^{n_3}\left(\frac{\rho_e}{\rho_c}\right)^{n_4}, \tag{8}$$

where $K = (4.1 \pm 0.3)\cdot 10^3$, $n_1 = -0.41 \pm 0.01$, $n_2 = -1.58 \pm 0.04$, and $n_3 = n_4 = 0.94 \pm 0.05$; in (8) and subsequently, the characteristic scale of the process l is taken as the minor radius r_1 of the toroid. Equation (8) was derived for the following ranges in the criteria: $\mathrm{Ra}^* = 0.3\cdot 10^{10}$–$2.0\cdot 10^{12}$, $\mathrm{Pr} = 1.6$–6.4, $m = 0.17$–0.90, $\rho_e/\rho_c = 0.5$–2.7, and $\mathrm{Fo}_* = 4.1\cdot 10^{-3}$–$3.7\cdot 10^{-2}$.

The following formula was derived for τ_{**}, the time when the external transient state ends:

$$\frac{\tau_{**}}{\tau_*} = K\left(x\frac{C''-C_p)}{C_p}\right)^n, \tag{9}$$

where $K = 0.15 \pm 0.01$ and $n = -0.39 \pm 0.01$; (9) was derived for the range $x(C''-C_p)/C_p = 1.0\cdot 10^{-5}$–$2.29\cdot 10^{-3}$, and the τ_* of (9) was calculated from (8). Also, (8) allows us to transform (9) to

$$\mathrm{Fo}_{**} = 620\,\mathrm{Ra}^{*-0.41}\,\mathrm{Pr}^{-1.58}(1-m)^{0.94}\left(\frac{\rho_e}{\rho_c}\right)^{0.94}\left[\frac{x(C''-C_p)}{C_p}\right]^{-0.39}. \tag{9a}$$

Formula (9a) was derived for the following ranges: $\mathrm{Ra}^* = 3\cdot 10^9$–$2\cdot 10^{12}$, $\mathrm{Pr} = 1.6$–6.4, $m = 0.17$–0.93, $\rho_e/\rho_c = 0.5$–2.7, $\mathrm{Fo}_{**} = 3\cdot 10^{-2}$–$8\cdot 10^{-2}$, and $x(C''-C_p)/C_p = 1.0\cdot 10^{-5}$–$2.29\cdot 10^{-3}$.

The following generalized relationship was derived for the temperature of the free surface:

$$\frac{\vartheta_s}{T_0} = \mathrm{Ho}_*^{n_1}\left(\frac{1-m}{m}\right)^{n_2}\bar{\tau}^{n_3}, \tag{10}$$

where $K = 1.29 \pm 0.02$, $n_1 = 0.76 \pm 0.01$, and $n_2 = 0.29 \pm 0.02$; the exponent n_3 in (10) is dependent on the volume filling factor m, as well as on the rate of heat transfer at the phase interface, which is defined by $x(C''-C_p)/C_p$, and also on the mode of the process in each stage. For instance, for $m \le 0.5$ (internal transient state) we have that $n_3 = 0.94 \pm 0.02$, while n_3 is given by the following expression for $m \ge 0.5$:

$$n_3 = K'\left[\frac{x(C'' - C_p)}{C_p}\right]^{n'}, \tag{11}$$

where $K' = 1.82 \pm 0.02$ and $n' = 0.093 \pm 0.004$ for the internal transient state, $\tau \le \tau_*$; whereas $K' = 1.29 \pm 0.01$ and $n' = 0.065 \pm 0.003$ for the external transient state, $\tau \ge \tau_*$. Equation (10) describes the surface temperature in states I and II for the following ranges in the parameters:

$$\mathrm{Ho}_* = 7.0 \cdot 10^{-4} - 5.2 \cdot 10^{-2},\ \frac{1-m}{m} = 0.1 - 5.1,\ \bar{\tau} = 0.2 - 5.0,$$

$$\frac{x(C'' - C_p)}{C_p} = 1.0 \cdot 10^{-5} - 2.29 \cdot 10^{-2}.$$

Formula (8) is used to derive τ_* for (10); Fig. 3 shows curves for (10).

It follows from (11) that in the internal transient state ($m \ge 0.5$) the range $x(C'' - C_p)/C_p = 1.0 \cdot 10^{-5} - 2.29 \cdot 10^{-3}$ corresponds to $n_3 = (0.62 \pm 0.03) - (1.03 \pm 0.03)$; if the limiting values of n_3 are substituted into (10) and (8) is used, we obtain the following expressions for the surface temperature of the liquid: for $n_3 = 0.62$,

$$\frac{\vartheta_s \lambda}{q r_1} = 4.2\,\mathrm{Ra}^{*-0.06}\,\mathrm{Pr}^{-0.22}\left(\frac{\rho_e}{\rho_c}\right)^{0.13}\mathrm{Fo}^{0,62}\left(\frac{q r_1}{\lambda T_0}\right)^{-0.24}(1-m)^{-0.16}\,m^{-1.05} \tag{10a}$$

and for $n_3 = 1.03$,

$$\frac{\vartheta_s \lambda}{q r_1} = 0.13\,\mathrm{Ra}^{*0.11}\,\mathrm{Pr}^{0.43}\left(\frac{\rho_e}{\rho_c}\right)^{-0.25}\mathrm{Fo}^{1.03}\left(\frac{q r_1}{\lambda T_0}\right)^{-0.24}(1-m)^{-0.54}\,m^{-1.05} \tag{10b}$$

Equations (10a) and (10b) relate to the following ranges: $\mathrm{Fo} = 3 \cdot 10^{-3} - \mathrm{Fo}_*$ [see (8)], $q r_1/\lambda T_0 = 0.1 - 3.2$, and $m = 0.50 - 0.90$.

In the external-transient state ($m \ge 0.5$), the range $x(C'' - C_p)/C_p = 1.0 \cdot 10^{-5} - 2.29 \cdot 10^{-3}$ corresponds to $n_3 = (0.62 \pm 0.02) - (0.87 \pm 0.02)$; we transform (10) with (8) to the following form for the limiting values of n_3: for $n_3 = 0.62$,

$$\frac{\vartheta_s \lambda}{q r_1} = 4.2\,\mathrm{Ra}^{*0.06}\,\mathrm{Pr}^{-0.22}\left(\frac{\rho_e}{\rho_c}\right)^{0.13}\mathrm{Fo}^{0.62}\left(\frac{q r_1}{\lambda T_0}\right)^{-0.24}(1-m)^{-0.16}\,m^{-1.05} \tag{10c}$$

and for $n_3 = 0.87$:

$$\frac{\vartheta_s \lambda}{q r_1} = 0.51\,\mathrm{Ra}^{*0.05}\,\mathrm{Pr}^{0.17}\left(\frac{\rho_e}{\rho_c}\right)^{0.10}\mathrm{Fo}^{0.87}\left(\frac{q r_1}{\lambda T_0}\right)^{-0.24}(1-m)^{-0.39}\,m^{-1.05} \tag{10d}$$

Equations (10c) and (10d) relate to the following ranges: $\mathrm{Fo} = \mathrm{Fo}_*$ [see (8)] $- \mathrm{Fo}_{**}$ [see (9a)], $q r_1/\lambda T_0 = 0.1 - 3.2$, and $m = 0.5 - 0.90$.

In the internal-transient state, small values of the filling number ($m \le 0.5$) gave the constant value $n_3 = 0.94 \pm 0.01$ for liquid oxygen and nitrogen for the range $x(C'' - C_p)/C_p = 2.7 \cdot 10^{-3} - 1.7 \cdot 10^{-2}$; this value for the exponent corresponds to the following expressionfor the surface temperature derived from (10) and (8):

$$\frac{\vartheta_s \lambda}{q r_1} = 0.28\,\mathrm{Ra}^{*0.07}\,\mathrm{Pr}^{0.28}\left(\frac{\rho_e}{\rho_c}\right)^{-0.16}\mathrm{Fo}^{0.94}\left(\frac{q r_1}{\lambda T_0}\right)^{-0.24}(1-m)^{-0.45}\,m^{-1.05} \tag{10e}$$

for $n_3 = 0.94$; (10e) applies for the range $\mathrm{Fo} = 10^{-3} - \mathrm{Fo}_*$ [see (8)], $q r_1/\lambda T_0 = 0.2 - 2.0$, and $m = 0.17 - 0.50$. Then (10) gives us the following conclusions. The rate of increase in the surface temperature itself increases with the heat-transfer rate at the phase boundary if the other conditions are unchanged, in accordance with (10); the exponent n_3 increases with the $x(C'' - C_p)/C_p$ of (11).

In the case of the high-boiling liquid (hexane), where the mass-transfer rate at the phase boundary is small [$x(C'' - C_p)/C_p \cong 10^{-5}$], we found a single temperature dependence for states I and II; the dependence on the Rayleigh and Prandtl numbers in (10a) is negative.

The cryogenic liquids (nitrogen and oxygen) gave $x(C'' - C_p)/C_p \cong 10^{-3}$, and for these we obtained different relationships for the internal and external transient states, as (10b) and (10d) show, which is due to the differences in the interactions between the heat- and mass-transfer processes at the different stages. The dependence on the Rayleigh and Prandtl numbers in (10b) and (10d) is positive.

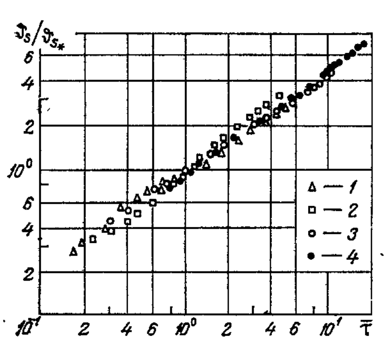

Fig. 2

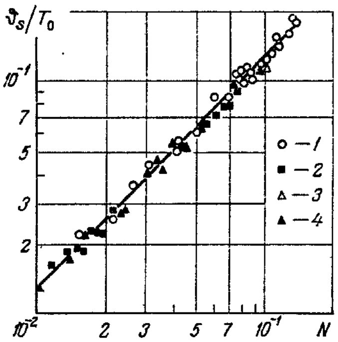

Fig. 3

Fig. 2. Surface temperature as a function of time normalized to the corresponding temperature and time for τ_*: 1) $q=254$ W/m^2, $r_1=0.072$ m, hexane, $x(C''-Cp)/Cp=0.30\cdot10^{-4}$, $m=0.80$; 2) $q=152$ W/m^2, $r_1=0.096$ m, oxygen, $x(C''-Cp)/Cp=0.60\cdot10^{-3}$, $m=0.86$; 3) $q=56.5$ W/m^2, $r_1=0.072$ m, nitrogen, $x(C''-Cp)/Cp=0.35\cdot10^{-3}$, $m=0.93$; 4) $q=28.3$ W/m^2, $r_1=0.072$ m, nitrogen, $x(C''-Cp)/Cp=0.45\cdot10^{-3}$, $m=0.90$.

Fig. 3. Generalized relationship for the free-surface temperature of a liquid: 1) oxygen, $r_1=0.096$ m; 2) hexane, $r_1=0.072$ m; 3) nitrogen, $r_1=0.096$ m; 4) nitrogen, $r_1=0.072$ m; $Ra=1.74\cdot10^9-0.65\cdot10^{12}$, $Fo=10^3-0.8\cdot10^{-1}$, $x(C''-Cp)/Cp=0.1\cdot10^{-4}-1.76\cdot10^{-2}$, $m=0.17-0.90$; $N=\bar{\tau}^{n_3}Ho_*^{0.76}[(1-m)/m]^{-0.29}$.

Small relative volumes of the cryogenic liquids ($m\leq0.5$) cause n_3 to be independent of $x(C''-Cp)/C_p$, and the constant value of (10e) applies, which is due to the rapid evaporation under these conditions. Also, (10e) has a positive dependence on the Rayleigh and Prandtl numbers.

From (10) we conclude that the sense of the effects expressed by the Rayleigh number is dependent on the rates of the mass-transfer processes and can alter from negative when the mass-transfer effect is weak to positive when mass transfer is rapid.

NOTATION

T, absolute temperature; T_0, initial temperature; T_c, critical temperature; $\vartheta=T-T_0$, change in temperature with time; Q, heat supplied; U, internal energy; C', specific heat of liquid on saturation line; C'', specific heat of saturated vapor; x, vapor dryness; C_p, specific heat of liquid at constant pressure; L, latent heat of evaporation; ρ', density on saturation line; ρ'', density of saturated vapor; S, surface area of vessel; l, characteristic dimension; Ra*, Rayleigh number; Fo, Fourier number; Pr, Prandtl number; $x(C''-C_p)/C_p$, flow-distribution criterion for the vapor system; k, coefficient of proportionality; n, exponent; V, volume of vessel; V_l, volume of liquid; g, acceleration of gravity; β, volume expansion coefficient; ν, kinematic viscosity; a, thermal diffusivity; λ, thermal conductivity.

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